

01. FUNCTIONS

LONG ANSWER QUESTIONS (7 MARKS)

01. Let $f : A \rightarrow B$, $g : B \rightarrow C$ be bijections. Then show that $\text{gof} : A \rightarrow C$ is a bijection.

Mar 09, May 06, 08, 10, 12; AP Mar 16

02. Let $f : A \rightarrow B$, $g : B \rightarrow C$ be bijections. Then show that $(\text{gof})^{-1} = f^{-1} \circ g^{-1}$.

Mar 06, 10, 11, 14, May 09, 11; TS Mar 16

03. Let $f : A \rightarrow B$ be a bijection. Then show that $\text{fof}^{-1} = I_B$ and $f^{-1} \circ f = I_A$.

Mar 07, 12, 15, 17, May 05, 07

04. If $f : A \rightarrow B$, $g : B \rightarrow A$ are two functions such that $\text{gof} = I_A$ and $\text{fog} = I_B$, then prove that 'f' is a bijection and $g = f^{-1}$.

Mar 01, May 03, Mar 08; TS May 15

05. I) If $f : \mathbb{R} \rightarrow \mathbb{R}$, $g : \mathbb{R} \rightarrow \mathbb{R}$ are defined by $f(x) = 4x - 1$ and $g(x) = x^2 + 2$, then find

(i) $(\text{gof})(x)$ (ii) $(\text{gof})\left(\frac{a+1}{4}\right)$ (iii) $\text{fof}(x)$ (iv) $\text{go}(\text{fof})(0)$

II) If $g : \mathbb{Q} \rightarrow \mathbb{Q}$ defined by $f(x) = 5x + 4$ for all $x \in \mathbb{Q}$. Show that 'f' is a bijection and find f^{-1} .

Mar 10; TS Mar 17

06. Let $f : A \rightarrow B$, I_A and I_B be identity functions on A and B respectively. Then show that $\text{fo}I_A = f = I_B \circ f$.

May 05, 08, Mar 13

VERY SHORT ANSWER QUESTIONS (2 MARKS)

07. If $f = \{(1, 2), (2, -3), (3, -1)\}$, then find (i) $2f$, (ii) $2 + f$, (iii) f^2 , (iv) $\sqrt{f}$. Mar 08, 12; AP Aug 22

08. If $f = \{(4, 5), (5, 6), (6, -4)\}$ and $g = \{(4, -4), (6, 5), (8, 5)\}$, then find TS Mar 17, May 22

i) $f + g$ ii) $f - g$ iii) fg iv) $\sqrt{f}$

09. If $f : \mathbb{R} - \{0\} \rightarrow \mathbb{R}$ is defined by $f(x) = x^3 - \frac{1}{x^3}$, then show that $f(x) + f\left(\frac{1}{x}\right) = 0$. AP May 22

10. If $A = \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$ and $f : A \rightarrow B$ is a surjection defined by $f(x) = \cos x$, then find B.

Mar & June 11, Mar 16; TS Mar 2017; AP Aug 22

11. If $A = \{-2, -1, 0, 1, 2\}$ and $f : A \rightarrow B$ is a surjection defined by $f(x) = x^2 + x + 1$, then find 'B'.

May 10; TS Mar 16, 17

12. If $f : \mathbb{R} \rightarrow \mathbb{R}$, $g : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3x - 1$, $g(x) = x^2 + 1$, then find

i) $\text{fof}(x^2 + 1)$ ii) $\text{fog}(2)$ iii) $\text{gof}(2a - 3)$.

13. If $f(x) = 2x - 1$, $g(x) = \frac{x+1}{2}$, for all $x \in \mathbb{R}$ are two functions, then find i) $\text{gof}(x)$ ii) $\text{fog}(x)$

AP & TS Mar 19

14. Find the inverse of the function $a, b \in \mathbb{R}$; $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = ax + b$ ($a \neq 0$). Mar 13

15. Find the inverse of the function $f: \mathbb{R} \rightarrow (0, \infty)$ defined by $f(x) = 5^x$. Mar 06, 11, 15

16. Find the inverse of the function $f: (0, \infty) \rightarrow \mathbb{R}$ defined by $f(x) = \log_2 x$.

17. Find the domain of the real function $\frac{2x^2 - 5x + 7}{(x-1)(x-2)(x-3)}$.

18. Find the domain of the real value function $f(x) = \frac{1}{6x - x^2 - 5}$.

19. Find the domain of the real value function $f(x) = \frac{\sqrt{3+x} + \sqrt{3-x}}{x}$. Mar 07

20. Find the domain of the real function $\sqrt{16 - x^2}$.

21. Find the domain of the real value function $f(x) = \sqrt{x^2 - 25}$. Mar 12

22. Find the domain of the real value function $f(x) = \frac{1}{\sqrt{x^2 - a^2}}$ ($a > 0$). Mar 15

23. Find the domain and range of the real valued function $f(x) = \sqrt{9 - x^2}$. TS Mar 15, 17

02. Mathematical Induction

LONG ANSWER QUESTIONS (7 MARKS)

01. Show that $2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \dots + \text{upto } n \text{ terms} = \frac{n(n^2 + 6n + 11)}{3}$, $\forall n \in \mathbb{N}$. May 06, Mar 13

02. Show that $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 + \dots + \text{upto } n \text{ terms} = \frac{n(n+1)(n+2)(n+3)}{4}$, $\forall n \in \mathbb{N}$.

TS Mar 15, 17

03. Show that $\forall n \in \mathbb{N}$, $\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \dots + \text{upto } n \text{ terms} = \frac{n}{3n+1}$. Mar 06, 11, May 11

04. Show that $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots + \text{upto } n \text{ terms} = \frac{n(n+1)^2(n+2)}{12}$, $\forall n \in \mathbb{N}$.

Mar 09, May 09, Mar 12; AP Mar 16

05. Show that $\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + \dots + \text{upto } n \text{ terms} = \frac{n}{24}(2n^2 + 9n + 13)$. Mar 05, 07, 14

06. Prove by Mathematical induction, for all $n \in \mathbb{N}$, $a + ar + ar^2 + \dots$ upto n terms $= \frac{a(r^n - 1)}{r - 1}$,
 $r \neq 1$. Mar 11; TS Mar 20; AP Mar 19
07. Use mathematical induction to prove the statement,
 $\left(1 + \frac{3}{1}\right)\left(1 + \frac{5}{4}\right)\left(1 + \frac{7}{9}\right) \dots \left(1 + \frac{2n+1}{n^2}\right) = (n+1)^2$. Mar 15; TS Mar 19
08. Show that $49^n + 16n - 1$ divisible by 64 for all positive integers 'n'.
 May 05, AP Mar 17, 20; TS Mar 18
09. Show that $3 \cdot 5^{2n+1} + 2^{3n+1}$ is divisible by 17, $\forall n \in \mathbb{N}$. May 08, 10, 12

03. Matrices

LONG ANSWER QUESTIONS (7 MARKS)

01. Show that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$.
02. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and $\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \neq 0$, then show that $abc = -1$. Mar 04, 14
03. Show that $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$. Oct 96, Mar 15
04. Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2$. Mar 12; TS Mar 18
05. Show that $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$. Mar 09; AP Mar 17
06. Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$. Mar & May 11
07. If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is a non-singular matrix, then show that A is invertible and $A^{-1} = \frac{\text{adj } A}{\det A}$. Mar 07, June 10; AP Mar 17; TS Mar 18

08. Solve the following simultaneous linear equation $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$ by using Cramer's rule. TS Mar 17; AP 16, 22
09. Solve the following simultaneous linear equation $3x + 4y + 5z = 18$, $2x - y + 8z = 13$, $5x - 2y + 7z = 20$ by using Cramer's rule AP Mar 15, 17, May 22, TS Mar 19
10. Solve the following simultaneous linear equation $x + y + z = 1$, $2x + 2y + 3z = 6$, $x + 4y + 9z = 3$ by using Cramer's rule AP & TS Mar 20
11. Solve the following simultaneous linear equation $x - y + 3z = 5$, $4x + 2y - z = 0$, $-x + 3y + z = 5$ by using Cramer's rule TS Mar 15, AP Mar 19, May 22
12. Solve the following simultaneous linear equation $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$ by using Matrix inversion method. AP Mar 15; TS Mar 19
13. Solve the following simultaneous linear equation $2x - y + 3z = 8$, $-x + 2y + z = 4$, $3x + y - 4z = 0$ by using Matrix inversion method. AP Mar 15; TS May 22
14. Solve the following simultaneous linear equation $3x + 4y + 5z = 18$, $2x - y + 8z = 13$, $5x - 2y + 7z = 20$ by using Matrix inversion method. AP Mar 15, May 22, TS Mar 19
15. Solve the following simultaneous linear equation $x - y + 3z = 5$, $4x + 2y - z = 0$, $-x + 3y + z = 5$ by using Matrix inversion method.
16. Solve the following simultaneous linear equation $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$ by using Gauss - Jordan method. AP Mar 16, 18
17. Solve the following simultaneous linear equation $3x + 4y + 5z = 18$, $2x - y + 8z = 13$, $5x - 2y + 7z = 20$ by using Gauss - Jordan method. TS Mar 15
18. Examine whether the following system of equation is consistent or inconsistent.
If consistent find the complete solution $x + y + z = 3$, $2x + 2y - z = 3$, $x + y - z = 1$ June 02
19. Examine whether the following system of equation is consistent or inconsistent.
If consistent find the complete solutions $x + y + z = 6$, $x - y + z = 2$, $2x - y + 3z = 9$. Mar 05, 11
20. By using Gauss-jordan method, show that the following system has no solution $2x + 4y - z = 0$, $x + 2y + 2z = 5$, $3x + 6y - 7z = 2$.

SHORT ANSWER QUESTIONS (4 MARKS)

21. If $\theta - \phi = \frac{\pi}{2}$, then show that $\begin{bmatrix} \cos^2 \theta & \cos \theta \cdot \sin \theta \\ \cos \theta \cdot \sin \theta & \sin^2 \theta \end{bmatrix} \begin{bmatrix} \cos^2 \phi & \cos \phi \cdot \sin \phi \\ \cos \phi \cdot \sin \phi & \sin^2 \phi \end{bmatrix} = \mathbf{0}$. Mar 04, May 09, 12
22. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then show that $(aI + bE)^3 = a^3I + 3a^2bE$. Mar 10, 15; TS Mar 16
23. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then show that $A^2 - 4A - 5I = \mathbf{O}$. AP Mar 16; TS Mar 17, May 22

24. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$, then find $A^3 - 3A^2 - A - 3I$, where I is unit matrix of order 3. TS Mar 19

25. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then find A^4 . TS Mar 17, 20

26. If $A = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -3 & 0 \\ 5 & 4 \end{bmatrix}$, then verify that $(AB)^T = B^T A^T$. Mar 13

27. If $A = \begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$, then find $A + A'$ and AA' . May 07; AP Mar 15,18,19

28. If $A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix}$, then find $(AB')'$. TS Mar 19

29. Find the value of x , if $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$. Mar 06; TS Mar 15

30. If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$, then show that $A^{-1} = A^T$. Mar 09, 14; AP Aug 22

31. If $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}$, then find the adjoint and inverse of A . Mar 05, 08

32. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$, then find A^{-1} . Mar 12; TS Mar 17; AP Aug 22

33. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$ then find $(A')^{-1}$.

34. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then show that the adjoint of A is equal to $3A^T$. Find A^{-1} . Mar 19

35. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then show that $A^{-1} = A^3$.

VERY SHORT ANSWER QUESTIONS (2 MARKS)

36. Construct a 3×2 matrix whose elements are defined by $a_{ij} = \frac{1}{2}|i - 3j|$. **TS Mar 15, Mar 17**

37. If $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & a-4 \end{bmatrix}$, then find x, y, z and a . **AP Mar 19**

38. If $\begin{bmatrix} x-1 & 2 & y-5 \\ z & 0 & 2 \\ 1 & -1 & 1+a \end{bmatrix} = \begin{bmatrix} 1-x & 2 & -y \\ 2 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, then find the values of x, y, z & a .

39. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 7 & 8 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -4 & -1 \end{bmatrix}$, then find $A + B$.

40. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$ and $2X + A = B$, then find 'X'. **Mar 95, 11, 13, 15**

41. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $3B - 2A$. **Mar 12; TS Mar 19; AP Aug 22**

42. If $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and $A^2 = O$, then find the value of k . **Mar 05, 14, 17, May 11, TS May 22**

43. If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, then find A^2 . **Mar 08, AP May 22**

44. Define trace of a matrix and find the trace of A , if $A = \begin{bmatrix} 1 & 2 & -1/2 \\ 0 & -1 & 2 \\ -1/2 & 2 & 1 \end{bmatrix}$. **June 10**

45. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$ is a symmetric matrix, then find the value of x .

Mar 05; AP Mar 16, TS May 22

46. If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ is a skew-symmetric matrix, then find the value of 'x'. **May 11**

47. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$ and $\det A = 45$, then find x . **Mar 03, 07, May 09**

48. If ' ω ' is a complex (non-real) cube root of unity, then show that $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$. **Mar 11, 14**

49. Find the adjoint and inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$.

AP Mar 18, TS May 22

50. Find the adjoint and the inverse of the matrix $\begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$.

Mar 09, 13

51. Find the rank of matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

Mar 08, June 10; TS Mar 18]

04. Addition of Vectors

SHORT ANSWER QUESTIONS (4 MARKS)

01. Let ABCDEF be a regular hexagon with centre 'O'. Show that

$$\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\overline{AD} = 6\overline{AO}.$$

May 09, 11; AP Mar 15, 16; TS Mar 17

02. If the points whose position vectors are $3\bar{i} - 2\bar{j} - \bar{k}$, $2\bar{i} + 3\bar{j} - 4\bar{k}$, $-\bar{i} + \bar{j} + 2\bar{k}$ and $4\bar{i} + 5\bar{j} + \lambda\bar{k}$ are coplanar, then show that $\lambda = \frac{-146}{17}$.

TS Mar 15

03. $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors. Prove that the following four points are coplanar $-\bar{a} + 4\bar{b} - 3\bar{c}$, $3\bar{a} + 2\bar{b} - 5\bar{c}$, $-3\bar{a} + 8\bar{b} - 5\bar{c}$, $-3\bar{a} + 2\bar{b} + \bar{c}$.

May 10, Mar 17

04. $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors. Prove that the following four points are coplanar $6\bar{a} + 2\bar{b} - \bar{c}$, $2\bar{a} - \bar{b} + 3\bar{c}$, $\bar{a} + 2\bar{b} - 4\bar{c}$, $-12\bar{a} - \bar{b} - 3\bar{c}$

TS Mar 15

05. In the two dimensional plane, prove by using vector methods, the equation of the line whose intercepts on the axes are 'a' and 'b' is $\frac{x}{a} + \frac{y}{b} = 1$.

June 00, 05, May 05

06. Show that the line joining the pair of points $6\bar{a} - 4\bar{b} + 4\bar{c}$, $-4\bar{c}$ and the line joining the pair of points $-\bar{a} - 2\bar{b} - 3\bar{c}$, $\bar{a} + 2\bar{b} - 5\bar{c}$ intersect at the point $-4\bar{c}$, when $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors.

AP Mar 19, 22; TS Mar 16, 22

07. If $\bar{a}, \bar{b}, \bar{c}$ are noncoplanar, find the point of intersection of the line passing through the points $2\bar{a} + 3\bar{b} - \bar{c}$, $3\bar{a} + 4\bar{b} - 2\bar{c}$ with the line joining the points $\bar{a} - 2\bar{b} + 3\bar{c}$, $\bar{a} - 6\bar{b} + 6\bar{c}$.

TS Mar 17, 19

VERY SHORT ANSWER QUESTIONS (2 MARKS)

08. Find the unit vector in the direction of vector $\bar{a} = 2\bar{i} + 3\bar{j} + \bar{k}$.

Mar 14; TS Mar 17

09. Let $\bar{a} = \bar{i} + 2\bar{j} + 3\bar{k}$ and $\bar{b} = 3\bar{i} + \bar{j}$. Find the unit vector in the direction of $\bar{a} + \bar{b}$.

TS Mar 16, TS Oct 21

10. Let $\bar{a} = 2\bar{i} + 4\bar{j} - 5\bar{k}$, $\bar{b} = \bar{i} + \bar{j} + \bar{k}$, $\bar{c} = \bar{j} + 2\bar{k}$. Find the unit vector in the opposite direction of $\bar{a} + \bar{b} + \bar{c}$.

Mar 09, 10, 12; AP Mar 15, 19, Aug 22; TS Mar 19, May 22

11. If $\overrightarrow{OA} = \bar{i} + \bar{j} + \bar{k}$, $\overrightarrow{AB} = 3\bar{i} - 2\bar{j} + \bar{k}$, $\overrightarrow{BC} = \bar{i} + 2\bar{j} - 2\bar{k}$ and $\overrightarrow{CD} = 2\bar{i} + \bar{j} + 3\bar{k}$, then find the vector $\overrightarrow{OD}$.
Mar 13; TS Mar 15, May 22
12. If the vectors $-3\bar{i} + 4\bar{j} + \lambda\bar{k}$ and $\mu\bar{i} + 8\bar{j} + 6\bar{k}$ are collinear vectors, then find λ and μ .
May 2010, Mar 14; AP Mar 18, TS May 22
13. If $\bar{a} = 2\bar{i} + 5\bar{j} + \bar{k}$ and $\bar{b} = 4\bar{i} + m\bar{j} + n\bar{k}$ are collinear vectors, then find the values of m and n .
June 11; AP Mar 16, 20, May 22; TS Mar 15, 17
14. If the position vectors of the points A, B and C are $-2\bar{i} + \bar{j} - \bar{k}$, $-4\bar{i} + 2\bar{j} + 2\bar{k}$ and $6\bar{i} - 3\bar{j} - 13\bar{k}$ respectively and $AB = \lambda AC$, then find the value of λ .
Mar 11; AP Mar 17, TS May 22
15. If α , β and γ are the angles made by the vector $3\bar{i} - 6\bar{j} + 2\bar{k}$ with the positive directions of the coordinate axes, then find $\cos \alpha$, $\cos \beta$ and $\cos \gamma$.
AP Mar 17
16. Find the vector equation of the line joining the points $2\bar{i} + \bar{j} + 3\bar{k}$ and $-4\bar{i} + 3\bar{j} - \bar{k}$.
Mar 08, 09, 10, 11; TS Mar 16, 18, Oct 21
17. Find the vector equation of the line passing through the point $2\bar{i} + 3\bar{j} + \bar{k}$ and parallel to the vector $4\bar{i} - 2\bar{j} + 3\bar{k}$.
Mar & May 07, June 10; AP Mar 15, 17, May 22; TS Mar 16, 17, May 22
18. Find the vector equation of the plane passing through the points $\bar{i} - 2\bar{j} + 5\bar{k}$, $-5\bar{j} - \bar{k}$ and $-3\bar{j} + 5\bar{k}$.
Mar 06, 11, 13; AP Mar 15, 17, 19
19. Find the vector equation of the plane passing through the points $(0,0,0)$, $(0,5,0)$ and $(2,0,1)$.
AP Aug 22

05. Product of Vectors

LONG ANSWER QUESTIONS (7 MARKS)

01. A line makes angles θ_1 , θ_2 , θ_3 and θ_4 with the diagonals of a cube. Show that $\cos^2 \theta_1 + \cos^2 \theta_2 + \cos^2 \theta_3 + \cos^2 \theta_4 = \frac{4}{3}$.
02. If $\bar{a} = \bar{i} - 2\bar{j} + 3\bar{k}$, $\bar{b} = 2\bar{i} + \bar{j} + \bar{k}$, $\bar{c} = \bar{i} + \bar{j} + 2\bar{k}$, then find $|(\bar{a} \times \bar{b}) \times \bar{c}|$ and $|\bar{a} \times (\bar{b} \times \bar{c})|$.
03. If $\bar{a} = 2\bar{i} + \bar{j} - 3\bar{k}$, $\bar{b} = \bar{i} - 2\bar{j} + \bar{k}$, $\bar{c} = -\bar{i} + \bar{j} - 4\bar{k}$ and $\bar{d} = \bar{i} + \bar{j} + \bar{k}$, then compute $|(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d})|$.
TS Mar 15
04. Find the shortest distance between the skew lines $\bar{r} = (6\bar{i} + 2\bar{j} + 2\bar{k}) + t(\bar{i} - 2\bar{j} + 2\bar{k})$ and $\bar{r} = (-4\bar{i} - \bar{k}) + s(3\bar{i} - 2\bar{j} - 2\bar{k})$, where s, t are scalars.
Mar 08, 09
05. If $A = (1, -2, -1)$, $B = (4, 0, -3)$, $C = (1, 2, -1)$ and $D(2, -4, -5)$, find the distance between AB and CD.
Mar 07, 12, 14
06. If $\bar{a}, \bar{b}, \bar{c}$ be three vectors. Then show that $(\bar{a} \times \bar{b}) \times \bar{c} = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{b} \cdot \bar{c})\bar{a}$.
Mar 04; AP Mar 15

SHORT ANSWER QUESTIONS (4 MARKS)

07. The vectors $\overrightarrow{AB} = 3\vec{i} - 2\vec{j} + 2\vec{k}$, $\overrightarrow{AD} = \vec{i} - 2\vec{k}$ represent adjacent sides of a parallelogram ABCD. Find the angle between the diagonals. TS Mar 18
08. Prove that the smaller angle θ between any two diagonals of a cube is given by $\cos \theta = \frac{1}{3}$
Mar & May 10, June 11
09. Show that the points $2\vec{i} - \vec{j} + \vec{k}$, $\vec{i} - 3\vec{j} - 5\vec{k}$ and $3\vec{i} - 4\vec{j} - 4\vec{k}$ are the vertices of a right angled triangle. Also find the other angles.
10. Show that the points $(5, -1, 1)$, $(7, -4, 7)$, $(1, -6, 10)$ and $(-1, -3, 4)$ are the vertices of a rhombus. Mar 13
11. Find the unit vector perpendicular to the plane passing through the points $(1, 2, 3)$, $(2, -1, 1)$ and $(1, 2, -4)$. May 10, TS May 22
12. Find a unit vector perpendicular to the plane determined by the points $P(1, -1, 2)$, $Q(2, 0, -1)$ and $R(0, 2, 1)$.
13. Find the area of the triangle whose vertices are $A(1, 2, 3)$, $B(2, 3, 1)$ and $C(3, 1, 2)$. Mar 08, 14
14. Let $\vec{a}$ and $\vec{b}$ be vectors, satisfying $|\vec{a}| = |\vec{b}| = 5$ and $(\vec{a}, \vec{b}) = 45^\circ$. Find the area of the triangle having $\vec{a} - 2\vec{b}$ and $3\vec{a} + 2\vec{b}$ as two of its sides. Mar 08, 18
15. If $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$, $\vec{b} = \vec{i} + \vec{j} - \vec{k}$ and $\vec{c} = \vec{i} - \vec{j} + \vec{k}$, then compute $\vec{a} \times (\vec{b} \times \vec{c})$ and verify that it is perpendicular to $\vec{a}$. TS Mar 19
16. For any two vectors $\vec{a}$ and $\vec{b}$. Then show that $(1 + |\vec{a}|^2)(1 + |\vec{b}|^2) = |1 - \vec{a} \cdot \vec{b}|^2 + |\vec{a} + \vec{b} + \vec{a} \times \vec{b}|^2$.
TS Mar 18
17. If $\vec{a} = 2\vec{i} + \vec{j} - \vec{k}$, $\vec{b} = -\vec{i} + 2\vec{j} - 4\vec{k}$ and $\vec{c} = \vec{i} + \vec{j} + \vec{k}$, then find $(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c})$.
Mar 09; AP Mar 15, 19; TS Mar 17
18. Find the vector having magnitude $\sqrt{6}$ units and perpendicular to both $2\vec{i} - \vec{k}$ and $3\vec{j} - \vec{i} - \vec{k}$.
19. If $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k}$ and $\vec{b} = \vec{i} + 4\vec{j} - 2\vec{k}$, then find $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$.
20. For any three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ prove that $[\vec{b} \times \vec{c} \quad \vec{c} \times \vec{a} \quad \vec{a} \times \vec{b}] = [\vec{a} \quad \vec{b} \quad \vec{c}]^2$.
21. Find the volume of the tetrahedron having the edges $\vec{i} + \vec{j} + \vec{k}$, $\vec{i} - \vec{j}$ and $\vec{i} + 2\vec{j} + \vec{k}$. May 09
22. Find the volume of the tetrahedron whose vertices are $(1, 2, 1)$, $(3, 2, 5)$, $(2, -1, 0)$ and $(-1, 0, 1)$.
May 07; TS Mar 15
23. If $\vec{a} = (1, -1, -6)$, $\vec{b} = (1, -3, 4)$ and $\vec{c} = (2, -5, 3)$, then compute $\vec{a} \times (\vec{b} \times \vec{c})$.

VERY SHORT ANSWER QUESTIONS (2 MARKS)

24. Find the angle between the vectors $\vec{i} + 2\vec{j} + 3\vec{k}$ and $3\vec{i} - \vec{j} + 2\vec{k}$.

Mar 10, 14; AP Mar 18; TS Mar 17, May 22

25. If $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{b} = 3\vec{i} - \vec{j} + 2\vec{k}$, then show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular to each other. May 11, Mar 15, TS May 22
26. If the vectors $\lambda\vec{i} - 3\vec{j} + 5\vec{k}$ and $2\lambda\vec{i} - \lambda\vec{j} - \vec{k}$ are perpendicular to each other, find λ . Mar 11, 14; TS Mar 16
27. If the vectors $2\vec{i} + \lambda\vec{j} - \vec{k}$ and $4\vec{i} - 2\vec{j} + 2\vec{k}$ are perpendicular to each other then find λ . TS Mar 15, 16
28. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then find the angle between $\vec{a}$ and $\vec{b}$.
29. $\vec{a} = \vec{i} - \vec{j} - \vec{k}$ and $\vec{b} = 2\vec{i} - 3\vec{j} + \vec{k}$, then find the projection vector of $\vec{b}$ on $\vec{a}$. Mar 91; AP Mar 17
30. Find the angle between the planes $\vec{r} \cdot (2\vec{i} - \vec{j} + 2\vec{k}) = 3$ and $\vec{r} \cdot (3\vec{i} + 6\vec{j} + \vec{k}) = 4$. AP Mar 20; TS Mar 15, 18, 20
31. If $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$ and $\vec{b} = \vec{i} - 3\vec{j} - 5\vec{k}$, then find $|\vec{a} \times \vec{b}|$. Mar 13
32. If $|\vec{a}| = 13$, $|\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 60$, then find $|\vec{a} \times \vec{b}|$. TS Mar 20
33. Let $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$ and $\vec{b} = 3\vec{i} + 4\vec{j} - \vec{k}$. If θ is the angle between $\vec{a}$ and $\vec{b}$, then find $\sin \theta$.
34. Find the unit vector perpendicular to the plane determined by the vectors $\vec{a} = 4\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = 2\vec{i} - 6\vec{j} - 3\vec{k}$. May 09, Jun 09, 10
35. If $\vec{a} = 2\vec{i} - 3\vec{j} + 5\vec{k}$, $\vec{b} = -\vec{i} + 4\vec{j} + 2\vec{k}$, then find $\vec{a} \times \vec{b}$ and unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. TS May 22
36. Find the vector area and area of the parallelogram having $\vec{a} = 2\vec{j} - \vec{k}$ and $\vec{b} = -\vec{i} + \vec{k}$ as adjacent sides.
37. Find the area of the parallelogram whose diagonals are $3\vec{i} + \vec{j} - 2\vec{k}$ and $\vec{i} - 3\vec{j} + 4\vec{k}$.
38. Find the area of the triangle having $(3\vec{i} + 4\vec{j})$, $(-5\vec{i} + 7\vec{j})$ as adjacent sides.

06. Trigonometry upto Transformations

LONG ANSWER QUESTIONS (7 MARKS)

01. If $A + B + C = 180^\circ$, then prove that $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$. AP Mar 19
02. If A, B, C are angles in a triangle, then prove that $\sin 2A - \sin 2B + \sin 2C = 4 \cos A \sin B \cos C$. TS May 22
03. If $A + B + C = 180^\circ$, then show that $\cos 2A + \cos 2B + \cos 2C = -4 \cos A \cos B \cos C - 1$.
04. If A, B, C are angles in a triangle, then prove that $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$.

05. If A, B, C are angles in a triangle, then prove that

$$\cos A + \cos B - \cos C = -1 + 4 \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \sin \frac{C}{2}.$$

May 06; TS Mar 19

06. If A, B, C are angles in a triangle, then prove that $\sin^2 A + \sin^2 B - \sin^2 C = 2 \sin A \sin B \cos C$.

AP Mar 19, Aug 22

07. If $A + B + C = \pi$, then prove that $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2} = 1 - 2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$.

08. If $A + B + C = 180^\circ$, then prove that $\cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} + \cos^2 \frac{C}{2} = 2 \left(1 + \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \right)$.

Mar 12, 15; TS Mar 15

09. If A, B, C are angles of a triangle, then prove that

$$\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} - \sin^2 \frac{C}{2} = 1 - 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}.$$

May 06, 11, AP May 22

10. If $A + B + C = 0$, then show that $\sin 2A + \sin 2B + \sin 2C = -4 \sin A \cdot \sin B \cdot \sin C$. AP Mar 19

11. If $A + B + C = \frac{\pi}{2}$, then prove that $\cos 2A + \cos 2B + \cos 2C = 1 + 4 \sin A \sin B \sin C$. AP Mar 20

12. $A + B + C = 2S$, then prove that

$$\cos(S - A) + \cos(S - B) + \cos(S - C) + \cos S = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}.$$

TS Mar 18

13. If $A + B + C = 2S$, then prove that $\sin(S - A) + \sin(S - B) + \sin C = 4 \cos \left(\frac{S - A}{2} \right) \cos \left(\frac{S - B}{2} \right) \sin \left(\frac{C}{2} \right)$.

14. If $A + B + C = 2S$, then prove that

$$\cos(S - A) + \cos(S - B) + \cos C = -1 + 4 \cos \left(\frac{S - A}{2} \right) \cos \left(\frac{S - B}{2} \right) \cos \frac{C}{2}.$$

TS Mar 17

SHORT ANSWER QUESTIONS (4 MARKS)

15. Show that $\cot \frac{\pi}{16} \cdot \cot \frac{2\pi}{16} \cdot \cot \frac{3\pi}{16} \cdot \cot \frac{4\pi}{16} \cdot \cot \frac{5\pi}{16} \cdot \cot \frac{6\pi}{16} \cot \frac{7\pi}{16} = 1$.

16. Prove that $\cos^2 \frac{\pi}{10} + \cos^2 \frac{2\pi}{5} + \cos^2 \frac{3\pi}{5} + \cos^2 \frac{9\pi}{10} = 2$.

AP Aug 22

17. Prove that $\cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{5\pi}{8} + \cos^2 \frac{7\pi}{8} = 2$.

18. If $A + B = \frac{\pi}{4}$, prove that

i) $(1 + \tan A)(1 + \tan B) = 2$

May 11

ii) $(\cot A - 1)(\cot B - 1) = 2$

Mar 07, May 09

iii) If $A - B = \frac{3\pi}{4}$, then show that $(1 - \tan A)(1 + \tan B) = 2$.

19. Prove that $\tan 70^\circ - \tan 20^\circ = 2 \tan 50^\circ$.

20. Prove that $\left(1 + \cos \frac{\pi}{10}\right) \left(1 + \cos \frac{3\pi}{10}\right) \left(1 + \cos \frac{7\pi}{10}\right) \left(1 + \cos \frac{9\pi}{10}\right) = \frac{1}{16}$.

AP Mar 15; AP & TS Mar 19

21. Prove that $\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8} = \frac{3}{2}$.

22. Prove that $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8} = \frac{3}{2}$.

AP May 16

23. Prove that (i) $\tan A + \cot A = 2 \operatorname{cosec} 2A$; (ii) $\cot A - \tan A = 2 \cot 2A$

Hence show that $\tan \theta + 2 \tan 2\theta + 4 \tan 4\theta + 8 \cot 8\theta = \cot \theta$.

VERY SHORT ANSWER QUESTIONS (2 MARKS)

24. If $\sin \theta = -\frac{1}{3}$ and θ does not lie in the third quadrant. Find the value of $\cos \theta$ and $\cot \theta$

Mar 13, TS Mar 19

25. If $\sin \theta = \frac{4}{5}$ and θ is not in the first quadrant, then find the value of $\cos \theta$.

AP Mar 19; TS Mar 17

26. If $3 \sin \theta + 4 \cos \theta = 5$, then find the value of $4 \sin \theta - 3 \cos \theta$.

May 12, AP May 22

27. If $a \cos \theta - b \sin \theta = c$, then show that $a \sin \theta + b \cos \theta = \pm \sqrt{a^2 + b^2 - c^2}$.

28. If $\tan 20^\circ = \lambda$, then prove that $\frac{\tan 160^\circ - \tan 110^\circ}{1 + \tan 160^\circ \tan 110^\circ} = \frac{1 - \lambda^2}{2\lambda}$.

29. Find $\sin 330^\circ \cdot \cos 120^\circ + \cos 210^\circ \cdot \sin 300^\circ$.

30. Find the value of $\cos 42^\circ + \cos 78^\circ + \cos 162^\circ$.

May 11

31. Find the value of $\sin^2 82\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$

TS Mar 17, 19

32. Find the value of $\cos^2 52\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$.

AP March 19

33. Prove that $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ = 4$.

AP Mar 17

34. Prove that $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \cot 36^\circ$.

Mar 11, 15; TS Mar 18

35. Show that $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = 0$.

TS Mar 18

36. If θ is not an integral multiple of $\frac{\pi}{2}$, prove that $\tan \theta + 2 \tan 2\theta + 4 \tan 4\theta + 8 \cot 8\theta = \cot \theta$.

37. What is the value of $\tan 20^\circ + \tan 40^\circ + \sqrt{3} \tan 20^\circ \tan 40^\circ$?

TS Mar 20

38. Prove that $\cos 48^\circ \cdot \cos 12^\circ = \frac{3 + \sqrt{5}}{8}$. TS Mar 17
39. Find the period of the functions $f(x) = \cos\left(\frac{4x+9}{5}\right)$ Mar 05, May 10, Mar 14
40. Find the period of the function $f(x) = \tan 5x$. TS May 22
41. Find the period of the functions $f(x) = \cos(3x+5)+7$
42. Find the period of the functions $f(x) = \tan(x+4x+9x+\dots+n^2x)$ (n is any positive integer). AP & TS Mar 15
43. Find a cosine function whose period is 7. Mar 13
44. Find a sine function whose period is $\frac{2}{3}$. AP May 22
45. Find the maximum and minimum values of the function $f(x) = 7\cos x - 24\sin x + 5$
46. Find the maximum and minimum values of the function $f(x) = 3\sin x - 4\cos x$. Mar 14
47. Find the maximum and minimum values of $3\cos x + 4\sin x$. TS Mar 16, 18
48. Find the maximum and minimum value of $f(x) = 5\sin x + 12\cos x - 13$.

07. Trigonometric Equations

SHORT ANSWER QUESTIONS (4 MARKS)

01. Solve $2\cos^2 \theta + 11\sin \theta = 7$.
02. Solve $2\cos^2 \theta - \sqrt{3}\sin \theta + 1 = 0$ and write the general solutions. May 09
03. Solve $1 + \sin^2 \theta = 3\sin \theta \cdot \cos \theta$. Mar 10
04. Solve $\sin x + \sqrt{3}\cos x = \sqrt{2}$. Mar 10
05. Solve the equation $\sqrt{3}\sin \theta - \cos \theta = \sqrt{2}$. Mar 18
06. Solve $\sqrt{2}(\sin x + \cos x) = \sqrt{3}$ and write the general solution. May 12, Mar 15
07. Solve $\sin 2x - \cos 2x = \sin x - \cos x$.
08. Solve the equation $\cot^2 x - (\sqrt{3} + 1)\cot x + \sqrt{3} = 0$, $\left(0 < x < \frac{\pi}{2}\right)$. Mar 12, 14

08. Inverse Trigonometric Functions

SHORT ANSWER QUESTIONS (4 MARKS)

01. Prove that $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$.

May 06, 10, 11, 15, Mar 11; AP Mar 15, 17; TS May 15, 18

02. Prove that $\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{3}{5} - \tan^{-1} \frac{8}{19} = \frac{\pi}{4}$.

03. Prove that $\sin^{-1} \frac{4}{5} + 2 \tan^{-1} \frac{1}{3} = \frac{\pi}{2}$.

Mar 10, TS Mar 15, AP May 16

04. Prove that $\sin^{-1} \left(\frac{4}{5} \right) + \sin^{-1} \left(\frac{7}{25} \right) = \sin^{-1} \left(\frac{117}{125} \right)$.

Mar 13, TS Mar 16

05. Prove that $\sin^{-1} \left(\frac{3}{5} \right) + \cos^{-1} \left(\frac{12}{13} \right) = \cos^{-1} \left(\frac{33}{65} \right)$.

AP Mar 16

06. Prove that $\cos^{-1} \left(\frac{4}{5} \right) + \sin^{-1} \left(\frac{3}{\sqrt{34}} \right) = \tan^{-1} \left(\frac{27}{11} \right)$.

May 13

07. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, then prove that $xy + yz + zx = 1$.

08. Prove that $\cos \left[2 \tan^{-1} \frac{1}{7} \right] = \sin \left[4 \tan^{-1} \frac{1}{3} \right]$

09. Show that $\cot \left(\sin^{-1} \sqrt{\frac{13}{17}} \right) = \sin \left(\tan^{-1} \frac{2}{3} \right)$.

TS Mar 17

09. Hyperbolic Functions

VERY SHORT ANSWER QUESTIONS (2 MARKS)

01. Prove that $\cosh^2 x - \sinh^2 x = 1$.

02. Show that $\tanh^{-1} \left(\frac{1}{2} \right) = \frac{1}{2} \log_e 3$.

Mar 05, 07, May 05, 07, Mar 15, May 15; AP Mar 17, 19

03. If $\sinh x = 5$ show that $x = \log_e (5 + \sqrt{26})$.

04. If $\sinh x = 3$, then show that $x = \log_e (3 + \sqrt{10})$.

TS Mar 16, AP May 22, Aug 22

05. For any $x \in \mathbb{R}$, prove that $\cosh^4 x - \sinh^4 x = \cosh(2x)$.

TS Mar 20

06. For any $x \in \mathbb{R}$, show that $\cosh 2x = 2 \cosh^2 x - 1$.

AP Mar 18

07. If $\cosh x = \frac{5}{2}$, find the value of (i) $\cosh(2x)$ and (ii) $\sinh(2x)$.

May 06, Mar 10, May 06, 11, Mar 11, 16; TS May 15, TS Mar 17, 19

08. If $\sinh x = \frac{3}{4}$, find $\cosh 2x$ and $\sinh 2x$

May 09, Mar 12, 14; TS May 16, 22

09. Prove that $(\cosh x - \sinh x)^n = \cosh(nx) - \sinh(nx)$, for any $n \in \mathbb{R}$

Mar 06, 07; TS Mar 15

10. Prove that $(\cosh x + \sinh x)^n = \cosh(nx) + \sinh(nx)$, for any $n \in \mathbb{R}$

TS Mar 18

10. Properties of Triangles

LONG ANSWER QUESTIONS (7 MARKS)

01. Show that $a \cos^2 \frac{A}{2} + b \cos^2 \frac{B}{2} + c \cos^2 \frac{C}{2} = s + \frac{\Delta}{R}$.

Mar 09; TS May 15

02. If $a : b : c = 7 : 8 : 9$, find $\cos A : \cos B : \cos C$.

AP May 15

03. If $\cot \frac{A}{2} : \cot \frac{B}{2} : \cot \frac{C}{2} = 3 : 5 : 7$, show that $a : b : c = 6 : 5 : 4$.

AP Mar 17

04. Prove that $\frac{\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}}{\cot A + \cot B + \cot C} = \frac{(a+b+c)^2}{a^2 + b^2 + c^2}$

TS May 22

05. If $a = (b - c) \sec \theta$, prove that $\tan \theta = \frac{2\sqrt{bc}}{b - c} \sin \frac{A}{2}$.

Mar 10, 11; AP Mar 18; TS May 16

06. If $a = (b + c) \cos \theta$, prove that $\sin \theta = \frac{2\sqrt{bc}}{b + c} \cos \frac{A}{2}$.

May 11; AP Mar 19

07. If $\sin \theta = \frac{a}{b + c}$, prove that $\cos \theta = \frac{2\sqrt{bc}}{b + c} \cos \frac{A}{2}$.

Mar 11, May 11, Mar 12; AP Mar & May 16, TS May 22

08. If P_1, P_2, P_3 are the altitudes drawn from vertices A, B, C to the opposite sides of a triangle respectively then show that i) $\frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} = \frac{1}{r}$ ii) $P_1 P_2 P_3 = \frac{(abc)^2}{8R^3} = \frac{8\Delta^3}{abc}$.

Mar 10; TS Mar 18

09. In a $\triangle ABC$ if $a = 13, b = 14, c = 15$, then show that $R = \frac{65}{8}, r = 4, r_1 = \frac{21}{2}, r_2 = 12, r_3 = 14$.

May 10, Jun 11, Mar 14; AP Mar 15, 16, Jun 16, Mar 19; TS Mar 17

10. If $r_1 = 2, r_2 = 3, r_3 = 6$ and $r = 1$ prove that $a = 3, b = 4$ and $c = 5$.

Mar 09; TS Mar 15, May 22

11. In $\triangle ABC$, if $r_1 = 8, r_2 = 12, r_3 = 24$, find a, b, c .

AP May 15, Mar 17; TS May 16

12. Show that $r + r_3 + r_1 - r_2 = 4R \cos B$.

Mar 13; AP Mar 18, May 22

13. In $\triangle ABC$, prove that $r_1 + r_2 + r_3 - r = 4R$.

Mar 06

14. If $r : R : r_1 = 2 : 5 : 12$, then prove that the triangle is right angled at A.

May 07 09

15. $\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{a^2 + b^2 + c^2}{\Delta^2}$.

TS Mar 17, 19

SHORT ANSWER QUESTIONS (4 MARKS)

16. Show that $a^2 \cot A + b^2 \cot B + c^2 \cot C = \frac{abc}{R}$. Mar 14
17. Show that $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \left(\frac{a^2 + b^2 + c^2}{2abc} \right)$. May 10, AP Aug 22
18. In ΔABC , if $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$, show that $C = 60^\circ$. TS Mar 17, 19
19. Prove that $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \frac{s^2}{\Delta}$. TS May 16
20. If $\cot \frac{A}{2}$, $\cot \frac{B}{2}$, $\cot \frac{C}{2}$ are in A.P., then prove that a , b , c are in A.P.
21. If $\sin^2 \frac{A}{2}$, $\sin^2 \frac{B}{2}$, $\sin^2 \frac{C}{2}$ are in H.P., then show that a , b , c are in H.P.
22. Prove that $\cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$. Mar 10, May 12; AP May 16; TS Mar 15, 18
23. Prove that $4(r_1 r_2 + r_2 r_3 + r_3 r_1) = (a + b + c)^2$.
24. Show that $(r_1 + r_2) \sec^2 \frac{C}{2} = (r_2 + r_3) \sec^2 \frac{A}{2} = (r_3 + r_1) \sec^2 \frac{B}{2}$. AP Mar 19

THE END